THE IMPACT OF OCKHAM'S READING OF THE *PHYSICS* ON THE MERTONIANS AND PARISIAN TERMINISTS

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Introduction

This essay begins with some observations about the translation of Aristotle's *Physics* that William of Ockham (ca. 1285-1347) used and about other sources that he consulted. In the more substantive part the essay examines Ockham's interpretation of Aristotle and the effect that his interpretation had on two Oxford Mertonians and some of the Parisian terminists, notably Nicholas Oresme. From these comparisons, the essay draws conclusions about Ockham's influence on later medieval Aristotelianism and on early modern philosophy of nature.

In this essay, we shall see authors who were influenced by Ockham in a positive and systematic way but also authors who adopted some of his ideas while departing from him on other ideas with which he is closely identified. In the latter case we can still speak of influence, but clearly it was selective. Regardless of these differences, we can see in Ockham's interpretation of Aristotle an especially influential approach that contributed to a departure from standard readings of Aristotle and to a major innovation in natural philosophy.[1]

The Latin Translation of Aristotle Used by Ockham

The scholars who edited Ockham's texts in natural philosophy did not agree completely among themselves about the authenticity of the texts that they edited. On the question of the translation of Aristotle used by Ockham, the evidence shows and scholars agree that he used the translation by James of Venice even though the

[1] For background on interpretations of Ockham, see A. Goddu, "Connotative Concepts and Mathematics in Ockham's Natural Philosophy," *Vivarium*, 31 (1993), 106-139.

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revision by William of Moerbeke had been available for fifty years.[2] First, the editors of the critical edition of Ockham's *Commentary on the Physics* based their conclusion for the most part on the *lemmata*, that is, the standard text divisions cited by Ockham.[3] Second, Ockham's commentaries often begin with an almost verbatim citation of the commented text; in those instances as well it is clear that he used the translation by James of Venice.[4] Third, Ockham read authors who had used the Moerbeke translation, but because they also cited only the *lemmata* for the most part, he could not have used the Moerbeke translation in their texts.[5] Even in those cases where Thomas Aquinas and Giles of Rome quoted the Moerbeke translation aside from the *lemmata*, Ockham preferred the version in James.[6] The editors of the *Commentary* cite one example from the *Auctoritates Aristotelis*, but this is the only example that has been identified.[7] Finally, the editors also pointed out that Ockham made one explicit mention of the Moerbeke version.[8]

From one example it is impossible to draw any significant conclusion, let alone a conclusion of general significance. The text illustrates a case where Ockham provided an interpretation of Averroes that is supported by the rendering in the Moerbeke version.[9] It is not clear that Ockham intended the comment as a criticism, but he suggested that Averroes could easily be misunderstood here, hardly

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[2] Guillelmus de Ockham, *Expositio in Libros Physicorum Aristotelis*, ed. V. Richter and G. Leibold, *Opera Philosophica* (henceforth *OP*), IV (St. Bonaventure, N.Y., 1985), 9*-12*. Cf. *Physica Vetus*, ed. F. Bossier and J. Brams, *Aristoteles Latinus*, VII, 1 and VII, 1.2 (Leiden / New York, 1990).

[3] Ockham, *Expositio*, *OP*, IV, 10*. See also V. Richter: "Wilhelm von Ockham als Expositor der aristotelischen Physik," in *Wissen, Glaube, Politik, Festschrift für Paul Asveld*, ed. W. Gruber et al., (Graz, 1981), 97-100, esp. 97.

[4] Ockham, *Expositio*, *OP*, IV, 10*-12*; Richter, "Wilhelm von Ockham," 98.

[5] Thomas Aquinas, Giles of Rome, Walter Burley, and many others used the Moerbeke version.

[6] Richter, "Wilhelm von Ockham," 98.

[7] Ockham, *Expositio*, I, 8, *OP*, IV, $98^{298\text{-}299}$. Cf. *Les Auctoritates Aristotelis*, ed. J. Hamesse (Louvain / Paris, 1974), 115^{86}.

[8] Ockham, *Expositio*, *OP*, V, 6*.

[9] Ockham, *Expositio*, VII, *OP*, V, $649^{9\text{-}19}$: "Dicit igitur primo quod alteratio non est secundum virtutes et malitias. Quod enim secundum virtutem non sit alteratio, probat. Quia virtus est perfectio; sed secundum perfectionem non est alteratio nec motus; ergo secundum virtutem non est alteratio nec motus. Maiorem probat Philosophus, 'quia tunc unumquodque maxime est perfectum et maxime secundum naturam, quando maxime attingit propriam virtutem. Sicut circulus tunc est maxime secundum naturam, quando maxime est circulus.' Igitur virtus est perfectio. Istam expositionem praetendit translatio nova cui concordat expositio Commentatoris quae est ista: In perfectione non est motus alterationis, igitur in virtutibus non est alteratio."

an unusual stance for Ockham to adopt.[10]

There is additional evidence that Ockham consulted the Moerbeke translations of the *Physics*, *Metaphysics*, and *Generation of Animals*.[11] Because of Ockham's silence on the choice of translation, scholars have been reluctant to attach any genuine philosophical significance to his selection. The editors of the *Commentary* also do not provide an explanation. The choice may have been a matter of convenience or a matter of indifference to Ockham. From his often harsh criticisms of predecessors and contemporaries it is clear that early in his academic career Ockham became frustrated by language that struck him as opaque and ontologically inflationary. As we shall see, even Aristotle did not escape his strictness in this regard. Although Ockham made excuses for Aristotle in those cases, it is clear that he regarded the careless use of language as the principal cause of confusion, misunderstanding, and error. Aristotle's text and the commentaries on them lent themselves to the sorts of problems that Ockham took great pains to diagnose and resolve, and there is little reason to think that one version was either less or more troublesome in that regard than another.

As for the sources that Ockham used, we must mention, first, the commentary by Averroes, whom Ockham regarded as a great authority. It is clear that Ockham used the translation by James closely with Averroes's commentary, presumably in the translation by Michael Scot, and it is likely that he was also familiar with the so-called "Oxford Gloss," but we are far from any definitive and authoritative conclusions on these matters.[12] His next favorite author-

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[10] Ockham, *Expositio*, VII, *OP*, V, 652^{38-48}: "Ideo dicendum est quod Commentator intelligit aliquid esse indivisibile quando non suscipit magis et minus, sicut ipse ponit de circulo. Unde assimilat virtutem circulo quod sicut unus circulus non est magis circulus, quia quando aliquid curvatur non est circulus nisi in fine curvationis et non ante, et propter hoc circulus consistit in indivisibili, hoc est, res non est circulus nisi in fine curvationis et non ante, sic perfectio rei est in fine et non ante motum. Ideo dicit perfectionem esse in indivisibili. Et sic est de virtute quod ante moderationem omnium passionum diversarum specierum vel generum, non est virtus. Sed tunc in fine est virtus. Ideo dicit Commentator virtutem indivisibilem quia unus non habet magis virtutem quam alius."

[11] These texts have been placed in the Appendix, especially examples 1-2, 4, 7-13, 15, and 17. See J. Brams and G. Vuillemin-Diem, "Physica Nova und Recensio Matritensis—Wilhelm von Moerbekes doppelte Revision der Physica Vetus," in *Aristotelisches Erbe im arabisch-lateinischen Mittelalter*, ed. A. Zimmermann, *Miscellanea Mediaevalia*, 18 (Berlin, 1986), 215-288, esp. 244-251.

[12] Cf. C. Burnett and A. Mendelsohn, "Aristotle and Averroes on Method in the Middle Ages and Renaissance: The 'Oxford Gloss' to the Physics and Pietro d'Alfeltro's

ity, though never cited by name, was Peter John Olivi.[13] Ockham cited the commentaries of Thomas Aquinas and Giles of Rome, although he nowhere mentioned them by name. Ockham seems to have considered Giles a major adversary, although he usually treated both Thomas and Giles with respect. By contrast, Ockham vigorously attacked the views of realists on universals, quantity, relation, and on the principles and causes of motion, and the views of so-called "moderns" on infinity, continuity, and final cause. It seems likely that the realists and moderns he attacked included Henry of Harclay, Walter Burley, Walter Chatton, John Duns Scotus, and possibly Thomas of Wylton.[14]

Despite the Franciscan affinity for Robert Grosseteste and despite Ockham's use of Grosseteste's *Commentary on the Posterior Analytics* and of Grosseteste's version of the *Nicomachean Ethics*, it seems that Ockham did not use Grosseteste's *Commentary on the Physics*.[15] As the *Summa Logicae* testifies, Ockham was very much influenced by Grosseteste's *Commentary on the Posterior Analytics*, so we may conclude that while Grosseteste influenced him on logic and method, he seems to have passed over Grosseteste's ideas on physics and metaphysics with respectful silence.

The point is, then, that as Ockham read Aristotle and Averroes in whatever translations, he found readings that needed correction of especially opaque language leaning towards inflationary ontological commitments that in his view endangered the coherence of philosophy.[16] As we turn now to Aristotle and Ockham's interpretation of

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Expositio Procmii Averroys," in *Method and Order in Renaissance Philosophy of Nature*, ed. D. A. Di Liscia, E. Kessler, and C. Methuen (Aldershot, 1997), 53-111, esp. 54 and 60-61. On the comparisons between Averroes and English commentaries, see S. Donati, "Physica I, 1: l'interpretazione dei commentatori inglesi della translatio vetus e la loro recezione del commento di Averroè," *Medioevo*, 21 (1995), 75-225.

[13]On Olivi, cf. Ockham, *Expositio*, *OP*, V, 6*.

[14]Compare Ockham, *Expositio*, *OP*, IV, 6* and *OP*, V, 12*.

[15] See R. C. Dales, ed., *Roberti Grosseteste Commentarius in VIII Libros Physicorum Aristotelis* (Boulder, 1963), v-xxvi. Cf. Ockham, *Expositio*, *Op*, IV, 6, and *OP*, V; and Ockham, *Summa Logicae* (henceforth *SL*) *OP*, I, ed. P. Boehner, G. Gál, and S. Brown (St. Bonaventure, 1974), 44*-45*.

[16] Inasmuch as a translation is an interpretation, we may surmise that Ockham's own translation, had he been able to read Greek and produce one of his own, would have been the "lean" Aristotle that Ockham somehow perceived in the translations and commentaries available to him. See the comprehensive analysis of Ockham's motives by J. Miethke, *Ockhams Weg zur Sozialphilosophie* (Berlin, 1969), 137-347, esp. 264-347. Compare with M. Adams, *William Ockham* (Notre Dame, 1987), II, 901-1347, esp. 961-1010; and with A. Goddu, "William of Ockham: Academic Theology and its Polemical Phase," in *The History of Franciscan Theology*, ed. K. Osborne (St. Bonaventure, 1994), 231-310.

Aristotle, we may restrict our attention to Ockham's philosophical program and to the methods appropriate to philosophical discourse.

Aristotle

There is an almost inevitably infinite regress in the study of a thinker and his relation to his predecessors. Historians often claim that we cannot understand Ockham, for example, without understanding his fellow Franciscan Duns Scotus[17] and several other contemporaries and predecessors. Sooner or later, most historical treatments of western thinkers lead us back to Plato and Aristotle. Even with these authors there is no shortage of differences in interpretation. Where do we start? There is no escaping the dilemma. This analysis begins with a very selective summary of Aristotle and relies on both his texts and on some modern defensible interpretations of his works. The themes and issues selected, however, are dictated by the peculiar emphasis and importance that Ockham attached to them.

In many of his works Aristotle's method is clearly dialectical in nature, which is to say that he began with reputable opinions on a matter, surveyed the puzzles engendered by these opinions, and then tried to resolve them. Even if Aristotle did not arrive at a perfect or completed system, however, he did aspire to produce a system in which knowledge could be grasped and organized as a totality. The totality of science would not have constituted a single, unified science, because Aristotle believed in dividing genuinely scientific knowledge by its objects into independent disciplines or sciences. Aristotle divided science into the theoretical, the practical, and the productive along with their appropriate truths, concepts, structures, methods, and even standards of rigor. There is, in short, no single set of truths from which all other truths derive or can be deduced. Although he showed a preference for axiomatic, deductive science, Aristotle did not present many of his own scientific works in such a systematic way.[18] Most commentators who did not get carried away by an exces-

[17] For example, R. Wood, *Ockham on the Virtues* (West Lafayette, Indiana, 1997), 12-13: "Few of Ockham's important philosophical or theological doctrines can be fully understood without reference to Scotus's view."

[18] The distinctions and consequences for philosophy derive, of course, from the *Nicomachean Ethics*, VI, 3, 1139b18-35 and VII, 1, 1145b1-7 and from *Metaphysics*, IV, 1, 1003a20-26. My presentation here is very much dependent on the wise remarks of J. Barnes, "Life and Work," in *The Cambridge Companion to Aristotle*, ed. J. Barnes (Cambridge, 1995), 22-26.

sively Platonic reading of Aristotle seem to have understood him in this way. Some, however, were far more rigorous and insistent and, therefore, drew consequences that stressed the individual conclusions, concrete *aporia*, and singular truths over the total, comprehensive, and systematic characteristics of Aristotle's thought.

In a similar fashion there has been a good deal of debate about Aristotle's theory of knowledge and the role of intuition. The *Posterior Analytics* spells out the structure of demonstrative sciences, but then leaves the reader puzzling over how we arrive at the first principles in the first place. There are both innatist and empiricist elements in Aristotle's account, so commentators have tended to split according to the emphasis that they place either on the outcome or on the process.[19] Ockham tended to be more interested in the process, and that tendency was consistent with his somewhat more empiricist leanings.

There is likewise no shortage of difference over what Aristotle meant by the categories, differences that arise from considerations of purpose and the texts in which the relevant comment is found. The full list of ten categories appears only in two works, the *Topics* and the *Categories*, although substance, quality, and quantity are always included.[20] In the *Metaphysics*, Aristotle says that "being" or "exists" has different meanings for different categories.[21] This is a doctrine that is not even found in the logical works, a doctrine that is crucial for understanding Ockham's interpretation of the categories. Although Ockham never fulfilled his promise to write a commentary on Aristotle's *Metaphysics*, Ockham relied on it in many of his other works. It is important to note here that Ockham's interpretation does have a basis in the Aristotelian text. Aristotle suggests that the existence of a substance is different from the existence of a quality, and things in other categories exist in ways that depend on or are derivative from the existence of a substance. Aristotle distinguished between primary and derivative senses of "exists" and "be."[22]

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[19] The reference here is, of course, to *Posterior Analytics*, II, 19. See also the remarks of R. Smith, "Logic," in *The Cambridge Companion to Aristotle*, 49-51.

[20] Aristotle, *Topics*, I, 9, 103b20-25 and *Categories*, 4, 1b25-2a4. See Smith, "Logic," 55-57.

[21] See J. Barnes, "Metaphysics," in *The Cambridge Companion to Aristotle*, 77-89, for a summary of the issues that follow.

[22] The following selections from Aristotle's *Metaphysics* corroborate these claims. Ockham cited the *Metaphysics* in several works. We do not know which versions he may have consulted. For comparison with a translation that Ockham might have consulted, I cite the Moerbeke translation, *Metaphysica*, ed. G. Vuillemin-Diem,

He also suggested that the derivative could be reduced to the primary, but not reduced to the point of eliminating the derivative senses.[23] Accordingly, commentators have differed about how reductive Aristotle intended the derivative kinds of categories to be understood. Are they real beings, or are they just modes of being, or do they have just derivative being, whatever that may mean?

Finally in our very selective summary of Aristotle's views, we turn to the role of the topics and of dialectical argument in his philosophy. Here again commentators have differed about the importance of probable reasoning and of the role of probable reasoning in arriving at the first principles of a science.[24] These are all questions and issues on which commentators had to take a position. Ockham's answers to these questions, especially about the categories and their reduction, influenced both his interpretation of Aristotle and his entire philosophical program.

Aristoteles Latinus, XXV, 3.2 (Leiden / New York / Cologne, 1995), V, 7, 1017a7-8, 102: "Ens dicitur hoc quidem secundum accidens illud uero secundum se." V, 7, 1017a19-27, 103: "QUE QUIDEM IGITUR secundum accidens esse DICUNTUR sic dicuntur aut eo quod eidem enti ambo insunt, aut quia enti illud inest, aut quia IPSUM EST cui inest de quo IPSUM predicatur. Secundum se uero esse dicuntur quecumque significant figuras predicatonis; quotiens enim dicitur, totiens esse significat. Quoniam ergo predicatorum alia quid est significant, alia quale, alia quantum, alia ad aliquid, alia facere, alia pati, alia ubi, alia quando, horum unicuique esse idem significant."

[23] Consider, for example, Aristotle's positive view about mathematical objects: *Metaphysica*, XIII, 3, 1077b12-30, 274-275: "Quod quidem igitur neque substantie magis corporum sunt, neque priora in essendo sensibilibus sed ratione solum, neque separata alicubi esse possibile, dictum est sufficienter. Quoniam autem neque in sensibilibus contingebat ipsa esse, manifestum quod aut totaliter non sunt aut modo quodam sunt, et propter hoc non simpliciter sunt. Multipliciter enim esse dicimus. Sicut enim et uniuersalia in mathematicis non de separatis sunt circa magnitudines et numeros, sed de hiis quidem, non in quantum autem talia qualia habere magnitudinem aut esse diuisibilia, palam quod contingit et de sensibilibus magnitudinibus esse et rationes et demonstrationes, non in quantum autem sensibiles sed in quantum tales. Sicut enim et in quantum mota solum multe rationes sunt sine eo quod quid unumquodque est talium et accidentium ipsis, et non necesse propter hec aut separatum aliquid esse motum sensibilium aut in hiis aliquam naturam esse separatam, sic et de motis erunt rationes et scientie, non in quantum mota autem, sed in quantum corpora solum; et iterum in quantum plana solum, et in quantum longitudines solum, et in quantum diuisibilia, et in quantum indiuisibilia habentia positionem, et inquantum indiuisibilia solum."

[24] See Smith, "Logic," 57-63. Compare with P. Slomkowski, *Aristotle's Topics*, *Philosophia Antiqua*, 74 (Leyden, 1997).

Ockham's Interpretation of Aristotle's Categories

We will comment on each of the issues above, but for the sake of brevity we begin with Ockham's understanding of the categories and the consequence that his understanding had for the division and relationship of the sciences, for intuition, and for dialectical method.[25]

We are barely a few lines into his major commentary on Aristotle's *Physics* before Ockham repeated the pattern that marks his earlier theological works by indicating his agenda. There are many confusing assertions in the literature that are caused by insufficient attention to logic. As Ockham saw it, these confusions could be avoided by consistently exercising a few simple precautions.[26]

To understand these precautions we must review briefly some of Ockham's principal doctrines in logic and how they were innovative and original.[27] Ockham's view that, typically, terms or words primarily signify things rather than concepts was not original, but his emphasis on the view that common terms signify individuals, not universal things, led to an original contribution in his theory of suppo-

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[25] For the most recent, incisive, and illuminating summary of Ockham's thought, see C. Panaccio, "William of Ockham," in *Routledge Encyclopedia of Philosophy*, ed. E. Craig, (London / New York, 1998), IX, 732-748.

[26] Ockham, *Expositio*, Prologue, *OP*, IV, 3-14. For example: $4^{35\text{-}39}$: "Sed antequam ad expositionem textus accedam, aliqua praeambula, sicut in principio *Logicae* feci, praemittam. Et quia forte *Expositio super Logicam* ad manus aliquorum non deveniet, qui tamen istam forte videbunt, idcirco aliqua ibidem dicta hic replicare addendo aliqua non pigebit." And $13^{87\text{-}101}$: "Sed diversa suppositio terminorum bene facit ad hoc quod de termino aliquod praedicatum vere praedicetur vel vere negetur. Unde ad hoc quod haec sit vera 'res mutabilis est subiectum vel illud de quo scitur' bene facit suppositio istius termini 'res mutabilis', non consideratio rei extra: nam si iste terminus 'res mutabilis' supponat simpliciter pro se, tunc est haec vera 'res mutabilis—hoc est hoc commune "res mutabilis"—est illud de quo aliquid scitur'; si autem supponat personaliter, tunc est falsa quia quaelibet singularis est falsa. Et ita diversa suppositio eiusdem termini bene facit ad hoc quod de eodem termino vere negetur aliquid et vere affirmetur. Nam si in ista 'homo est species' 'homo' supponat simpliciter, haec est vera; et si in ista 'homo non est species' idem terminus supponat personaliter, illa est etiam vera. Sed quod illa res quae est extra, propter unam considerationem meam sit mutabilis et propter aliam considerationem meam sit immutabilis, est simpliciter falsum et asinine dictum."

[27] The next several paragraphs are heavily dependent on S. Brown, "A Modern Prologue to Ockham's Natural Philosophy," in *Sprache und Erkenntnis im Mittelalter*, ed. A. Zimmermann, *Miscellanea Mediaevalia*, 13, 1 (Berlin, 1981), 107-129. I am indebted to Paul Spade for helping to make my summary less ambiguous. Cf. A. Goddu, "Ockham's Philosophy of Nature," in *The Cambridge Companion to Ockham*, ed. P. Spade (Cambridge, 1999), 143-167. In the present version I quote the sources more fully than was possible in the Cambridge volume.

sition. Because he rejected any common reality existing in things, he rejected theories of simple supposition that hold a common reality as the significate of a common term. When a supposing term stands for its significate (a thing), it has personal, not simple, supposition, according to Ockham, because the only true things are individual things. When a term has simple supposition, it stands for the concept, but the concept is, typically, not the significate of the term because a term that stands for its significate stands for true things, not for mental intentions.

In Ockham's account, then, individuals are the only significates; universals are concepts or words. Universal concepts or words can signify only individual things. A term that stands for a universal concept supposits non-significatively, and therefore it has simple supposition. When a term stands for individuals, it supposits significatively, and therefore it has personal supposition.

Ockham's move produced a reversal in supposition theory, and its cash value, so to speak, is the effect that reversal had on his interpretation of the categories. Substance terms and abstract quality terms are absolute terms that supposit significatively, and therefore there are real things these terms can refer to in personal supposition. Science in its most strict Aristotelian sense is of universals, and so science cannot be of things because there is no universal reality. Science is of concepts, then, but real science is a science in which the concepts stand for things, and rational science is a science in which the concepts stand for other concepts. Hence, natural philosophy is a real science, whereas logic is a rational science.

Concrete quality terms and terms in all of the other categories are connotative terms that supposit significatively for one thing primarily and another thing secondarily, and therefore there are real things these terms can refer to in personal supposition. Connotative terms do not signify things distinct from individual substances and inhering qualities, that is to say, the terms in all categories other than substance and quality signify something real but not a distinct thing existing subjectively in singular substances like individual inhering qualities. A relation such as 'similarity' signifies something real, namely, that something (an essential characteristic, property, quality, or whatever) that two things have that makes them similar, not similarity itself as an entity inhering subjectively in them. A term in the category of quantity signifies that something is quantified or numbered, not quantity itself as an entity inhering subjectively in the things quantified or numbered.

Ockham's doctrine of connotation has caused scholars many problems.[28] There remain important differences in interpretation, but I focus on the consequences of his doctrine for natural philosophy and science. Every science is of concepts, not things. From Ockham's point of view most of the mistakes in philosophy and science have arisen from two fundamental mistakes: (1) the assumption that science is of things leading to fruitless searches for common realities and universal essences, and (2) the assumption that every term has some thing corresponding to it, leading to the postulation of superfluous entities. As Ockham put it, "This, however, is an abusive way of dealing with terms and leads people away from the truth."[29]

Connotative terms require descriptions that cannot be reduced to a single term, and so such concepts and words are often deceiving. Part of the difficulty with Ockham's doctrine is that he never found a general way of analyzing propositions that use connotative terms. What apparently defeated him was the fact that philosophers use such terms in many different and even equivocal ways. It is impossible to produce an exhaustive general statement, and so Ockham accepted the unavoidable, though not without complaint. He would have preferred that abstract terms in categories other than substance and quality not be used at all. He admitted that he too had to speak as others do, using their language and the very abstract terms he wanted to avoid, but he insisted that positing totally distinct realities corresponding to them can be avoided. The job of the philosopher,

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[28] Cf. Ockham, *SL*, I, 9-10, 38-62; II, 11-14; III-1, 31; III-2, 33-34; III-3, 26; III-3, 29, *OP*, I. See also Ockham, *Scriptum in Librum Primum Sententiarum: Ordinatio*, d. 2, qq. 4-8, *Opera Theologica* (henceforth *OT*), II, ed. G. Gál, S. Brown, G. Etzkorn, and F. Kelley (St. Bonaventure, 1970), 99-292. Here is not the place to rehearse the problems but, judging from the most recent studies on connotation and related problems, we have to say that scholars are far from a consensus. In my opinion, most of the analytically rigorous accounts are operating from a too narrow base, textually and philosophically. Here is a sample of the most recent literature: Y. Zheng, "Metaphysical Simplicity and Semantical Complexity of Connotative Terms in Ockham's Mental Language," *The Modern Schoolman*, 75 (1998), 253-264; P. Spade, "Three Versions of Ockham's Reductionist Program," *Franciscan Studies*, 56 (1998), 347-358; J. Beckmann, "Ockham, Ockhamismus, und Nominalismus: Spuren der Wirkungsgeschichte des Venerabilis Inceptors," *Franciscan Studies*, 56 (1998), 77-95; J. Boler, "Ockham on Difference in Category," *Franciscan Studies*, 56 (1998), 97-113; J. Boler, "Accidents in Ockham's Ontological Project," *Franciscan Studies*, 54 (1994-1997), 79-97. See also C. Panaccio, *Les mots, les concepts et les choses* (Montreal / Paris, 1991), and A. Goddu, "Connotative Concepts." By consulting the further citations provided by these authors, the reader can acquire a comprehensive view of the issues.

[29] Brown, "A Modern Prologue," 121; cf. *SL*, I, 51, *OP*, I, 171.

then, is to provide translations for terms that can be misleading. Many of Aristotle's expressions and examples should not be taken literally but figuratively.[30] With such cautions repeated tirelessly Ockham attempted to understand Aristotle's use of concepts and terms in his account of nature and the physical world.

Ockham's Interpretation of Quantity and Mathematical 'Entities'

With respect to quantity and mathematics, Ockham denied the existence of quantity as a thing distinguishable from a thing that is quantified. A quantity is not a real thing distinct from substance and quality. When a subject undergoes some real change, for example, if its length changes, then the change is real, and a proposition expressing that quantitative change expresses a real truth or a reality about the subject. Likewise, he denied the existence of mathematical entities. He placed quantity terms and mathematical terms under connotative terms and concepts. What motivated Ockham to adopt this approach to quantity and mathematics?[31]

Modern science is inconceivable without mathematics, but what relation do mathematics and nature have in Aristotle's conception? As Aristotle maintained, mathematics abstracts from the things that physics considers.[32] Ockham concluded, then, that mathematics and physics are distinct sciences because they consider diverse conclusions having diverse subjects or diverse predicates. Should they consider the same conclusion in such a way that the conclusion pertains essentially to both sciences, then it is possible that a part of physics may be subordinate to some part of mathematics, or the other way around.[33] With respect to the middle sciences such as perspective, harmony, and astronomy, it was his view that they belong to physics

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[30] Brown, "A Modern Prologue," 125-129.

[31] Ockham, *Expositio*, I, *OP*, IV, 53-62. For example, $55^{53\text{-}57}$: "Propter praedicta posset aliquis credere quod sit de intentione Philosophi et Commentatoris ponere nullam substantiam esse quantitatem sed esse rem aliam. Sed istis non obstantibus tenendum est quod non est de mente Philosophi ponere quantitatem esse aliam rem a substantia et qualitate, sicut super Logicam est ostensum."

[32] Aristotle, *Metaphysics*, VI, 1, 1026a7-32.

[33] Ockham, *Expositio*, II, *OP*, IV, 256-267. For example, $261^{75\text{-}81}$: "Et si considerent aliquando eandem conclusionem, tunc illa pars aliquando est eadem et est tam pars mathematicae quam physicae—et hoc si illa conclusio per se pertineat ad utramque scientiam—, aliquando etiam potest contingere quod una pars scientiae physicae potest subalternari alicui parti mathematicae et aliquando e converso; nec est hoc aliquod inconveniens."

more than they do to mathematics. Perspective and astronomy consider geometrical lines as physical, and they draw conclusions about attributes that are a part of natural philosophy. The sounds studied in harmony, however mathematical the relationships, are also physical. These sciences, then, are less abstract than the purely mathematical sciences. Ockham's reading here tends to support the standard Aristotelian view that mathematics is subordinate to natural philosophy.

On the other hand, in his theory of connotation, Ockham developed an interpretation of mathematical entities that subordinates ontological considerations to what we might call a more pragmatic conception of mathematics. It is a conception that makes mathematics into a language or into another tool of analysis. As such, mathematics can be used to clarify any subject matter. His interpretation seems to have influenced many authors of his time to use mathematics in sciences that do not involve measurement, where mathematics is a kind of theoretical formalism that enables us to resolve thorny questions about qualitative contraries, time, place, and the like.

Ockham rendered the connotative terms and concepts used in natural philosophy into sets of explaining propositions. That is to say, such terms must always be thought of as shorthand for complex propositions that describe or summarize two or more facts and conditions or some relationship that the terms signify. In some instances he denied altogether that the terms refer to anything existing.

For example, mathematical terms such as 'point', 'line', and 'surface' are fictive. The explaining propositions that render such terms are properly conditional not assertoric in form. Euclid's fifth postulate provides a good example. It should be rendered thus: "If two parallel lines were extended to infinity, they would never intersect."[34] In Ockham's view, we need not assume the real existence of lines or suppose that they can actually be extended to infinity. Mathematicians do not have to assume their real existence to employ them usefully.

To view mathematics as an inventory of objects was inflationary, according to Ockham. That his view was congenial to many authors follows from mathematical applications that were dictated by formal and not empirical considerations. Bradwardine's analysis that pro-

[34] Ockham, *Expositio*, III, *OP*, IV, 584-585; esp. $585^{26\text{-}29}$: "Sicut mathematici non accipiunt talem propositionem categoricam 'lineae rectae aeque distantes in infinitum protensae numquam concurrunt', sed accipiunt talem propositionem 'si lineae rectae aeque distantes in infinitum protenderentur, numquam concurrerent'."

duced his famous law of motion proceeds from the mathematics of ratios, not from empirical considerations.[35] If an analysis begins from some simplifying assumption, several results can be derived whether empirically verified or not.

Although some fourteenth-century mathematicians were probably mathematical realists (Bradwardine, for example), Ockham and others shifted attention away from mathematics as discussion about objects and entities to mathematics as a language and a formalism that can be interpreted in many ways and applied to all disciplines.[36] He refuted the Aristotelian prohibition against *metabasis*, the prohibition against using mathematics to represent other categories of being. If we think of mathematics primarily as a language and not as a category of being, then the prohibition loses its force.[37]

As an example of Ockham's criticism of the focus on ontological issues, we may consider an application of his refutation of Aristotle's prohibition against *metabasis*.

In the *Commentary on the Physics*, VII, 5-6, he discussed at some length the question whether motions of different kinds are comparable.[38] The Aristotelian analysis recognizes obstacles or places a priori considerations in the way of comparison. Aristotle saw words being used equivocally, for example, when we say that a pencil, wine, and a musical note are sharp, we are obviously using the term 'sharp' differently.[39] Aristotle regarded 'circular' and 'rectilinear' as qualities whose properties or attributes cannot be compared.[40] This fol-

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[35] See M. Clagett, *The Science of Mechanics in the Middle Ages* (Madison, 1959), 206: "Needless to say, the discussion of variations of quality and velocity by the Oxford schoolmen (and their medieval successors) was almost entirely hypothetical and *not rooted in empirical investigations*. In fact, in medieval kinematics as well as dynamics all of the quantitative statements relative to pretended physical variables are in terms of general proportionality expressions; *and the proportionality constants, which can only be determined by experiment, are never found*."

[36] On Bradwardine's realism, see A. G. Molland, "An Examination of Bradwardine's Geometry," *Archive for History of Exact Sciences*, 19 (1978), 113-175, esp. 144-145.

[37] Ockham, *Expositio*, II, *OP*, IV, 256-267.

[38] Ockham, *Expositio*, VII, 5, *OP*, V, 665: "[Capitulum 5 Qui motus ad invicem sunt comparabiles?]." *Expositio*, VII, 6, *OP*, V, 670: "[Capitulum 6 Quid requiritur ad hoc quod aliquid sit alteri comparabile?]."

[39] Ockham, *Expositio*, VII, 6, *OP*, V, 670-671: "Circa primam partem primo proponit quid requiritur ad hoc quod aliqua sint comparabilia dicens quod quaecumque non sunt aequivoca, sunt comparabilia, et quae sunt aequivoca, non sunt comparabilia. Et illud declarat per exemplum. Quia enim acutum dicitur aequivoce de stylo et vino et ultima corda sive de pneumate; acutum enim praedicabile de istis, diversam habet definitionem."

[40] Ockham, *Expositio*, VII, 6, *OP*, V, 671: ". . . de motu recto et circulari, isti motus secundum velocitatem non sunt comparabiles, . . ."

lows from his insistence that the two must have no specific difference from each other either in themselves or in that in which they are manifested. Ockham provided further examples of invalid comparisons, yet in several instances he diagnosed the problem as one of ambiguity requiring further clarification. A dog and a horse are distinct substances, yet we can compare the whiteness of a dog with the whiteness of a horse. Following on that example, he then argued that if you imagine someone walking in a circle, then that individual's motion in a circle is comparable to the motion of someone walking in a straight line, and hence that circular motion is comparable with rectilinear motion.[41] As we all know from a modern example, in track events that require runners to sprint in curved lanes, the starting positions of the runners are staggered to insure that all of the runners sprint exactly the same distance.

Ockham pointed out that a rope that is straight is not longer than the same rope when it is rolled up into a coil. In this sense, a circular line is longer, shorter, or equal to some straight line.[42] Such examples led him to ask about the conditions that must be satisfied for something to be comparable to another. Here is the point where he directed his questioning of (attack on) the Aristotelian assumptions behind the analysis. If Aristotle insisted that the movements and the tracks or trajectories in question are equivocal, then are we to add that the means of locomotion also establish specific differences in the motions themselves, for example, walking with feet or flying with wings? We may dismiss such distinctions and confine ourselves to the formation of the track and say that the equal velocity means traversing the same distance in equal time; only the "sameness" must be specific in the case of the track and (consequently) in the case of the movement.

Ockham resolved such problems through an analysis of language. If abstract terms convey inherent qualities, then such general terms are not comparable. But if abstract terms such as 'curved', 'slow',

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[41]Ockham, *Expositio*, VII, 6, *OP*, V, 684: "Tamen aliquo modo concedi potest quod motus partis circulariter moti, quae pars secundum se totam mutat locum et pertransit spatium, est comparabilis motui recto."

[42]Ockham, *Expositio*, VII, 5, *OP*, V, 670: "Ad modum quo dicimus quod una corda involuta, quamvis tota iaceat in loco parvo, est longior quam alia cuius partes protensae secundum rectum magis distant. Unde non dicimus quod corda primo protensa secundum rectum et postea involuta, est propter hoc brevior. Unde si de aliquo recto fiat postea curvum vel sphaera, non dicimus quod erit brevius quam prius: immo dicimus quod est aeque longum. Et hoc vocabulo 'longitudo' sic accepto, possumus dicere quod linea circularis est longior, brevior et acqualis lineae rectae."

'unequal', 'swift', and the like, do not convey such things, then they are comparable, because they do not convey specifically different things. In other words, if the unique most specific definitions express the nominal definition, and if the nominal definition is predicated of both terms, then the terms are comparable.[43]

Ockham went on to interpret Aristotle as distinguishing between ontological claims and the use of nominal definitions which do permit comparisons that would otherwise be ontologically incomparable. He interpreted Aristotle's text as a discussion about the use of terms and about genus and species. In the end, however, he rejected Aristotle's claim that the motion of a body on a curve cannot be compared with its motion on a straight line. For example, the sun moves more rapidly than a heavy or a light body because the sun covers a greater distance in a day than a heavy body does.[44]

Ockham's Interpretation of Motion and the Infinite

Aristotle's definition of nature posits the notions of change and motion as essential parts of the definition.[45] Anyone ignorant of the causes of change and motion is ignorant of nature and of the nature of motion.

Motion, argued Ockham, is not a really distinct thing, distinct from the body undergoing motion and from the successive places that the body occupies. The change that a body undergoes is not distinct from the body in the act of change. Such a change refers to the potential acquisition of a qualification or of a part that the body now lacks or is in the process of acquiring. A sudden change refers to the

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[43]Ockham, *Expositio*, VII, 6, *OP*, V, 677: "Alia sunt nomina quorum abstracta non important res tales. Cuiusmodi sunt curvum, tardum, inaequale, velox et huiusmodi. Et talia, ad hoc quod sint comparabilia, non oportet quod importent res specie differentes, sed oportet quod nomen secundum quod debet fieri comparatio, secundum unicam definitionem specialissimam exprimentem quid nominis praedicetur de eis. Ita quod, si una definitio sufficienter in particulari et distincte exprimens quid nominis praedicetur de uno et non de reliquo, illa secundum tale nomen non sunt comparabilia. Si autem secundum talem definitionem praedicetur de utroque, et illud nomen recipit comparationem, oportet quod secundum tale nomen sint illa comparabilia."

[44]Ockham, *Expositio*, VII, *OP*, V, for example, 684: "Tamen aliquo modo concedi potest quod motus partis circulariter moti, quae pars secundum se totam mutat locum et pertransit spatium, est comparabilis motui recto. Sic enim dicimus quod sol velocius movetur quam grave vel leve quia maius spatium pertransit in die quam grave."

[45]Aristotle, *Physics*, III, 1, 200b12-14.

acquisition of a form all at once. To refer to change and motion as distinct entities is to commit the mistake of taking a connotative term for an absolute term. 'Motion' is a connotative term, and the misuse of that term illustrates the dangers of using abstract terms as a kind of shorthand for more complex expressions and events.

Some commentators on Ockham have seen in his denial of the existence of motion a denial that motion itself can change, so that all motion would have to be uniform and there could be no acceleration or deceleration.[46] But Ockham's point was different. He recognized that bodies undergo different kinds of motion, and that each requires a description adequate to the kind of motion the body undergoes. Bodies that undergo a uniform acceleration, for example, do not require us to suppose that the motion exists independently of the body in motion and that the disembodied motion itself is increasing in a uniform way, but merely that the body is moving with uniform acceleration.

On infinity, Ockham held with Aristotle that we can talk of things as being infinite only in the potential sense, for example, a thing is potentially divisible to infinity. Where he perhaps introduced a twist is in his explanation of the potentially infinite. If a continuous thing can be divided to infinity, then it must have an actually infinite number of parts otherwise we would be able to complete its division. Aristotle posited a potentially and not actually infinite number of parts, argued Ockham, not because there are not present an actually infinite number of parts, but because all of the parts are not and cannot actually be separated. If they could actually be separated, then they would be finite, not infinite. To say that a thing is potentially infinitely divisible requires us to suppose that it has an actually infinite number of parts, otherwise we would be able to complete the division, and if we could complete the division, then its parts would be finite.[47]

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[46] For example, A. Maier, *Zwei Grundprobleme der scholastischen Naturphilosophie, Studien zur Naturphilosophie der Spätscholastik*, II (Rome, 1955^2), 74-78.

[47] Ockham, *Expositio*, III, 14, 4, *OP*, IV, 411-587. For example, 584$^{17-22}$: "Hic tamen sufficit ad propositum quod Philosophus non intendit quod magnitudo sit subiectum motus tamquam cuiusdam rei exsistentis in ea et totaliter distinctae ab ea, nec est causa infinitatis in motu tamquam cuiusdam rei distinctae secundum se totam a magnitudine et motu, nec similiter motus est subiectum temporis, sicut posterius ostendetur." Also *Expositio*, VI, *OP*, V, 562-563$^{47-51}$: "Item, medietas alicuius totius exsistit actualiter in rerum natura, ergo eadem ratione medietas illius medietatis exsistit actualiter et per consequens quaelibet medietas exsistit actualiter; sed medietates sunt infinitae, quia non sunt in aliquo numero certo; igitur infinitae partes exsistunt

The relevance to natural science can be derived from the above examples. As Aristotle himself emphasized, spatial magnitudes, motion, and time are necessarily finite or infinite. Inasmuch as the science of nature is concerned with spatial magnitudes, motion, and time, it is concerned with the finite and infinite. Ockham tried to eliminate all of the ontological considerations and worries and replace them with talk about concepts. The clarification of concepts, in turn, would provide solutions to a wide array of pseudo-problems. His views on number and infinity represent one of several metalinguistic approaches to such problems. The approaches mark a shift in the interpretation and application of mathematics, and as such they are linked with later developments in acoustics, astronomy, and mechanics.

We turn now in a summary fashion to Ockham's ontological program and a final comparison with Aristotle. There is hardly a text where Ockham's effort to reduce talk about particular existing things to individual substances and qualities does not generate severe problems in ontology, logic, and semantics. Generally speaking, we may say that his strategy for resolving problems generated by his reductive ontological program was semantic. That in the realm of particulars only absolutes exist does not mean that nonabsolutes are nonexistent, that is, that it cannot be said of them that they are, exist, or occur. What this means in his ontology, however, is harder to say.

In my view, Ockham was a reductivist in ontology but not an eliminative reductivist. Categories other than substance and quality have a derivative being, but how we can coherently characterize their derivative being remains a disputed topic. There are many conditions, states, and relations in which absolute things exist. Propositions that assert such conditions, states, and relations do not assert the existence of any thing beyond the absolute, but they do assert ways in which substances and qualities exist.

Ockham resisted thinking of ways or modes of being as accidents, and hence as real things really distinct from their subjects. He treated them syntactically as adverbs or by way of circumlocution. The realist program is demonstrably impossible, he thought, because it is inflationary in ways that lead to contradiction, to vicious circularity, or to infinite regress. Why does the assertion of the existence of modes generate such problems? In some instances, the answer is that the existence of some entities is mind-dependent in ways that the judgment of the existence of some object is not. An object is a partial cause of the apprehension of the object, but some things like place

and time are not apprehended directly; they are inferred. The only genuinely independent existents in the realm of particulars are ones that cause direct apprehensions of them as existing. All other apparent existents are dependent on absolutes and hence are not really distinct from them. This is not to say, however, that time and place are subjective. Terms in some categories are transcategorical or they follow from the apprehension of multiple beings. For example, relative terms are comparative or complementary, following upon the apprehension and subsequent comparative judgment about at least two existing things, or following upon the same thing apprehended at two different times or places. In other words, there is an objective character or something objective about our experience of time and place and about some of our apprehensions that lead to comparative judgments. The being of the last eight categories must be construed, then, in a transcendental or transcategorical fashion, because their being is derived from a substance or a quality.[48]

Ockham limited real sciences to those that have real objects apprehended and intuited directly by the senses and the intellect. In emphasizing the processes of apprehension, judgment, habit formation, and generalization, he challenged inflationary accounts of knowledge and metaphysics. In that sense his view was reductionist. Because objects, subjects and predicates of propositions, and conclusions are individual and so diverse, however, the real sciences cannot be reduced to one deductive system. He definitely took Aristotle as primarily a puzzle-solver, a philosopher who surveyed likely opinions, critiqued them, and retained what was plausible.[49] Ockham, in addition, saw no problem with the possibility that the

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actualiter." Finally, *Expositio*, VI, *OP*, V, 564[89-97]: "Dicendum est secundum intentionem Philosophi quod partes infinitae quarum quaelibet secundum se totam est extra aliam et quarum quaelibet est accipere primam, non componunt aliquod finitum. Sed partes infinitae actualiter exsistentes quarum nulla est prima nec quaelibet earum secundum se totam est extra aliam, quas dicit Philosophus esse in potentia quia non sunt quaedam tota separatim exsistentia, bene possunt componere aliquod finitum, immo valde parvum finitum. Et huiusmodi sunt partes continui actualiter tamen exsistentes, quamvis non sint a toto divisae."

[48] This account blends together three independent interpretations as the most adequate to date: Beckmann, "Ockham," 87-89; Panaccio, *Les mots*, 31-35, 48-49, 56-61, 240-247; and Goddu, "Connotative Concepts," 118-123.

[49] For Ockham's understanding of probable premises and conclusions, see A. Goddu, "The Dialectic of Certitude and Demonstrability According to William of Ockham and the Conceptual Relation of his Account to Later Developments," in *Studies in Medieval and Natural Philosophy, Studi e testi*, I ed. S. Caroti and J. Murdoch (Città del Castello, 1989), 95-131, esp. 114-122.

results could be arranged in a number of coherent ways depending on the principles selected.[50] In a way this view of Aristotle was compatible with the detachment of numerous questions and problems from contexts in Aristotle and with the discussion of issues independently of other Aristotelian principles.

As a reader of Aristotle, Ockham believed Aristotle meant to distinguish the beings of substances and qualities from the being of the other categories. The being of all of the other categories is a derivative being. They are in a way, but not in the same way in which substances and qualities are said to be. The failure to observe that distinction strictly produced an inflationary ontology, according to Ockham. In his program the emphasis on this reading also freed mathematics to play a far more significant role in the analysis of conceptual problems. The cash value of that move, however, is found in the works of some Mertonians and Nicholas Oresme. It is fair to conclude that Ockham's Aristotelianism had a profound impact on the transition from a qualitative natural philosophy to a more, though cautious, quantitative conception of natural philosophy.

In the very brief sections that fellow, we provide just sufficient evidence to support the conclusion about the consequences of Ockham's interpretation of Aristotle for later medieval and early modern philosophy of nature.

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[50] Ockham, *Expositio*, Prologus, *OP*, IV, 9-10[98-120]: "Ex istis etiam patet quod quaerere quid est subiectum logicae vel philosophiae naturalis vel metaphysicae vel mathematicae vel scientiae moralis, nihil est quaerere, quia talis quaestio supponit quod aliquid unum sit subiectum logicae et similiter philosophiae naturalis, quod est manifeste falsum, quia nihil unum est subiectum totius sed diversarum partium diversa sunt subiecta. Unde quaerere quid est subiectum philosophiae naturalis, est simile quaestioni qua quaereretur quis est rex mundi. Quia sicut nullus est unus rex mundi, sed unus est rex unius regni et alter alterius, sic est de subiectis diversarum partium scientiae talis; nec plus scientia quae est talis collectio, habet unum subiectum quam mundus habet unum regem vel quam unum regnum habet unum comitem. Tamen pro dictis aliquorum auctorum qui videntur assignare unum subiectum talium scientiarum, est sciendum quod non intendunt quod aliquid sit proprie subiectum primum totius, sed intendunt dicere quod inter omnia subiecta diversarum partium est aliquod unum primum aliqua primitate, et aliquando unum est primum una primitate et aliud est primum alia primitate. Sicut in metaphysica primum inter omnia subiecta primitate praedicationis est ens, sed primum primitate perfectionis est Deus. Similiter in philosophia naturali primum subiectum primitate praedicationis est substantia naturalis vel aliquid aliud, et primum primitate perfectionis est homo vel corpus caeleste vel aliquid tale. Et hoc intendunt auctores per talia verba, et nihil aliud."

The Mertonians

The accomplishments of the Mertonians in natural philosophy are now well known. The dependence of William of Heytesbury and John Dumbleton on Ockham has not received the attention it deserves, however. Here we focus exclusively on Ockham's doctrines and the use that Heytesbury and Dumbleton made of them.

The sources of Mertonian techniques are still a matter of some debate, but they seem to have involved several strands: discussions about the division of the sciences and the relation between mathematics and natural philosophy; debates about the nature of quantity and its relation to substance and quality; the tendency to replace the view of motion as a qualitative accident with entirely quantitative and relational (or functional) considerations based not on empirical evidence or measurement but on mathematical consistency or coherence and on ontologically reductionist critiques of earlier views; discussion of kinematics and of variations in the intensity of a quality or essence; and competing conceptions of medicine as an art and medicine as a science.[51]

The keys to understanding the development of metalinguistic and mathematical analysis by Heytesbury are Ockham's theory of connotation and the liberation of mathematics from ontological constraints.[52] Ockham's insistence on comparisons between different kinds of motion provided the logical ground for laws that state the equivalence of accelerated and decelerated motions to uniform motions of the same duration.[53] Those laws of motion, in turn, provided models for rules of equivalence between different latitudes of quality.[54] The claim here is not that Ockham made such compari-

[51] For some helpful guides, see S. Drake, "Medieval Ratio Theory vs. Compound Medicines in the Origins of Bradwardine's Rule," *Isis*, 64 (1973), 67-77; M. McVaugh, "Arnald of Villanova and Bradwardine's Law ," *Isis*, 58 (1967), 56-64; and A. C. Crombie, "Quantification in Medieval Physics," *Isis*, 52 (1961), 143-160.

[52] This assertion is not meant as a denial of other influences on Heytesbury. See C. Wilson, *William Heytesbury: Medieval Logic and the Rise of Mathematical Physics* (Madison, 1956), 18-21, for the influences exercised on Heytesbury by mathematicians, philosophers, and theologians. Compare with the incisive comments of E. J. Dijksterhuis, *The Mechanization of the World Picture*, tr. C. Dikshoorn (London / New York, 1961), II, sections 117-118, 186-188.

[53] As stated by Heytesbury among others. See Wilson, *William Heytesbury*, 21.

[54] Wilson, *William Heytesbury*, 18-25. Compare Clagett, *Science*, 241-242. Clagett collated Brugge, Stadsbibliotheek, lat. 497; Brugge Stadsbibliotheek, lat. 500; Città del Vaticano, Vat. lat. 2136, with the edition of Bonetus Locatellus, Venice 1494 of *Regule Solvendi Sophismata Guilelmi Heytesberi* : "Est autem circa intensionem et

sons possible, for the historical evidence shows that mathematicians made comparisons without the permission of philosophers. What he did contribute, however, were arguments that challenged Aristotelian objections and to that extent provided logical and philosophical support for and sanction of comparisons and techniques of analysis that led to the precise fourteenth-century concept of latitude or range of degrees.

The terms that are typically found in Mertonian treatises are 'intensity', 'latitude', and 'extension'. Although these terms did not have the same meanings for all fourteenth-century authors, for the proponents of the additive theory the latitude of a quality was identified with the intensity of a quality at a given point. The proponents of the additive theory of qualitative increase thought of increase in terms of incremental degrees or grades, a notion that was subsequently distinguised from extensive magnitude to explain, for example, why the addition of the same degree of heat has different effects in different subjects. Hence, latitudes are virtually identical with degrees, and both are regarded as divisible. That is, a degree, according to this view, is not represented by a discrete arithmetical unit but by a continuous geometrical line.[55]

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remissionem motus localis advertendum, quod motum aliquem intendi vel remitti dupliciter contingit: uniformiter scilicet aut difformiter. Uniformiter enim intenditur motus quicunque, cum in quacunque equali parte temporis, equalem acquirit latitudinem velocitatis. Et uniformiter etiam remittitur motus talis, cum in quacunque equali parte temporis, equalem deperdit latitudinem velocitatis. Difformiter vero intenditur aliquis motus, vel remittitur, cum maiorem latitudinem velocitatis acquirit vel deperdit in una parte temporis quam in alia sibi equali. Iuxta illud sufficienter apparet, quod cum latitudo motus seu velocitatis sit infinita, non est possibile aliquod mobile ipsam uniformiter acquirere in aliquo tempore finito. Et quia quilibet gradus velocitatis per latitudinem tantummodo finitam distat a non gradu, seu termino privativo totius latitudinis, qui est quies; ideo a quiete ad gradum quemcunque datum, contingit aliquod mobile uniformiter intendere motum suum; et consimiliter, a gradu dato contingit motum uniformiter remittere ad quietem; et universaliter, a quocunque gradu ad quemcunque alium contingit utramque mutationem fieri uniformen."

[55] Heytesbury, *Regule*, in Clagett, *Science*, 279-280: "Cum ergo quelibet latitudo sit quedam quantitas, et universaliter sicut in omni quanto medium equaliter distat ab extremis, ita cuiuslibet latitudinis finite medius gradus equaliter distat ab utroque extremorum, sive illa duo extrema sint duo gradus, aut unum illorum fuerit aliquis gradus, et alterum omnino privatio sive non gradus illius. Sed sicut iam ostensum est, dato gradu aliquo sub quo etiam infiniti alii continue proportionales, quilibet ad sibi proximum signetur, equalis erit differentia seu latitudo inter primum et secundum, suum scilicet subduplum, sicut latitudo composita ex omnibus differentiis seu latitudinibus inter omnes gradus residuos, sequentes videlicet duos primos; ergo equaliter precise et per equalem latitudinem distabit ille gradus secundus subduplus sibi ad primum duplum, ab illo duplo, sicut distabit idem secundus a non gradu seu

As Ockham recognized, absolutes (substances and qualities) are subject to change. The logical problem was how to describe or "denominate" the conditions under which an individual is qualified by the attribute that the term connotes. 'Denomination' refers to the application of these logical techniques to contexts involving change and motion. 'Denomination' is the determination of how and what to name a subject undergoing change and how to characterize the change. The medieval philosophers who developed these techniques were not describing actual measurements and did not develop techniques of empirical measurement. In that sense it is correct to say that the analyses do not derive from real cases. The evidence in support of such a reading of Heytesbury is primarily of two kinds: 1) that his discussions are concerned with fundamental principles of analysis, and 2) that the rules generated are always applied by way of example to the sorts of questions or assertions that students encountered in natural-philosophical or theological contexts. In other words, the resolution of fundamental problems requires a discussion of terms before one can decide questions about the things to which the terms are applied. The exercises in which students engaged were intended to provide them with examples of how to resolve analytical problems in philosophy and theology.[56]

Ockham's denial of the reality of mathematical entities was the logical ground for Heytesbury's efforts to describe manifold circumstances of change and variation in mathematical terms related to the conventions of ordinary speech.[57] The ironic result was a liberation of mathematics from the limitations of actual experience and the creation of a tool that would make it possible to analyze actual

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ab extremo opposito illius magnitudinis date. Et sicut universaliter probatur de omni latitudine incipiente a non gradu et terminata in aliquem gradum finitum, continente etiam gradum aliquem et subduplum et subquadruplum et sic in infinitum, quod eius gradus medius est precise subduplus ad gradum ipsam terminantem. Unde non solum est hoc verum de latitudine velocitatis motus incipientis a non gradu, sed etiam de latitudine caliditatis, frigiditatis, luminis, et aliarum similium qualitatum, et consimiliter argui poterit et probari." See also E. Sylla, "Medieval Concepts of Latitudes of Forms: The Oxford Calculators," *Archives d'histoire doctrinale et littéraire du moyen âge*, 40 (1973), 223-283, esp. 251-256.

[56] See E. Sylla, "Medieval Quantifications of Qualities: The 'Merton School'," *Archive for History of Exact Sciences*, 8 (1971), 9-39; and J. Murdoch, "'*Scientia mediantibus vocibus*': Metalinguistic Analysis in Late Medieval Natural Philosophy," in *Sprache und Erkenntnis im Mittelalter*, ed. A. Zimmermann, *Miscellanea Mediaevalia*, 13, 1 (Berlin, 1981), 73-106. For a recent, clear statement of the achievement here, see E. Grant, *The Foundations of Modern Science in the Middle Ages* (Cambridge, 1996), 89-104 and 148-152.

[57] Wilson, *William Heytesbury*, 24.

experience more precisely. As he denied the reality of time, motion, and instants, Heytesbury accepted the common manner of speech (as Ockham also did, albeit reluctantly) and admitted that time, motion, and instants are measured by instants that are only imaginable fictions.[58]

John Dumbleton, indeed, was less interested in the use of mathematics to resolve strictly logical problems and more concerned with understanding physical reality; nevertheless, the horizon of his application of mathematical techniques was defined by Aristotelian qualitative considerations.[59] There is little evidence that Dumbleton relied on observable measurements, which indicates that his mathematical approach was not limited by physical measurement, let alone by empirically verifiable rules.[60] Dumbleton too followed Ockham in rejecting the ontological anxieties of other Aristotelians. He clearly

[58] Wilson, *William Heytesbury*, 51-56, shows how the analysis amounts to regarding the instantaneous as a limit. Wilson, 179-180, n. 55, cites "*De incipit et desinit*" from *Regule Solvendi Sophismata*, ed. Bonetus Locatellus, Venice 1494, fol. 26ra: "Quid autem instans sit in rerum natura, et qualiter continue sit aliud instans et aliud, longe est alterius perscrutationis, et multa figmenta falsa admittit modus loquendi hominis de instanti, tempore, et motu propter breviloquium et mentis conceptum facilius exprimendum, quia in rerum natura non est aliquid quod est instans ut instans nec tempus ut tempus aut motus ut motus; sicut nihil est Sortes prout ipse est homo albus nec aliquid est Plato prout ipse est disputaturus cras, aut prout ipse debet hodie respondere. Sed ista sunt impertinentia proposito. Et ideo ad ultimum quod ibidem propositum fuerat, scilicet quod multa incipiunt esse et etiam incipient et desinent esse, quorum nullum erit in instanti, dicitur negando illam propositionem iuxta communem modum loquendi quia omne quod est, est in instanti, eo quod illud instantance mensurat instans sive sit tempus vel motus aut etiam instans." For the explicit interpretation as only "imaginable," Wilson cites fol. 94^{vb}: "Talia successiva (motus et tempus) vel sunt aliqua existentia in rerum natura vel non sunt, sed tantum imaginabilia. . . ."

[59] See E. Sylla, "The Oxford Calculators and Mathematical Physics: John Dumbleton's Summa Logicae et Philosophiae Naturalis, parts II and III," in *Physics, Cosmology and Astronomy, 1300-1700: Tension and Accommodation*, ed. S. Unguru (Dordrecht / Boston, 1991), 129-161.

[60] Sylla's claims, *Oxford Calculators*, 151-152, about Dumbleton's concern with physical reality are ambiguous. Dumbleton's concern was derivative; he was commenting on Aristotelian texts, not researching empirical cases. Sylla cites texts on 160-161 from Cambridge, Peterhouse, lat. 272. I consulted Cambridge, Gonville and Caius College, lat. 499/268, f. 48ra: "Pro isto distinguendum quod tempus potest accipi dupliciter pro materia vel forma vel composito ex hiis ymaginarie vel ratione"; and f. 51^{vb}: "Pro isto dico sic: quod non maior ratio artat ponere puncta quam qualitates indivisibiles intensive, et ideo ymaginatio est solum que facit puncta"; and f. 52ra: "Pro isto concedendum est iuxta modum loquendi quod C tempus fuit et tamen negandum est quod B linea fuit. Et causa est quod tempus est tale quod <si> cum suis terminis fuerit in actu, ipsum non est nisi apud ymaginationem. Sed de aliis quarum partes manent simul, ita sunt cum suis terminis."

applied the techniques to clarify Aristotelian concepts and language about qualities and change, motion and time.[61] We will return to the significance of these efforts after a brief look at Parisian natural philosophy.

Oresme and Parisian Natural Philosophy

Historians have tended to distort the relation of Ockham to the Parisian terminists. In correcting earlier exaggerations, they have left us to wonder if Ockham exercised any influence on Parisian natural philosophy.[62] The Parisian terminists did not follow Ockham in his program of ontological reduction, but their approaches to problems in natural philosophy took their point of departure, in part, from his views on connotative concepts and his conception of mathematics. In my view, many of the puzzles about the characterizations of mathematics as "imaginary" and "conceptualist" can be resolved by focusing on the logical status of mathematical concepts. Some Parisian natural philosophers followed Ockham in regarding quantitative mathematical terms as fictive and, therefore, connotative, and they followed him in liberating mathematics from ontological constraints.

As we noted above, modern scholars have been unable to reach a consensus on Ockham's theory of connotation and its relation to his ontology. The reaction of some Parisian natural philosophers to him suggests that they encountered a similar difficulty. They, like Ockham, rejected extreme or strong realist interpretations of mathematical entities, motion, place, and time. Like him, they understood the terms and concepts suppositing for such 'entities' to be connotative. Like some modern interpreters, the Parisians read Ockham's theory as eliminatively reductive. Accordingly, the Parisians adopted interpretations that were less reductive though still less robust than those of the realists. It is clear that some Parisians were groping for a moderate position that would deny of some of the things for which connotative terms stand the status of an entity while not denying the reality

[61] As Sylla, 142, also acknowledges: "His mathematics is not mathematics as a separate science, but only mathematics as a tool for clarifying physical concepts and making them more exact and comprehensive."

[62] For example, Anneliese Maier's brilliant, ground breaking studies and corrections of Pierre Duhem's exaggerations were flawed by her notoriously unsympathetic view of Ockham. See Maier's *Studien zur Naturphilosophie der Spätscholastik*, 5 volumes (Rome, 1952-1968).

of the states or conditions that the connotative terms describe. Bodies do really move, yet 'motion' does not require us to suppose that motion is an entity distinct from the body in motion.

The texts of the Parisian natural philosophers provide some clues about the specific problems that led them to reject Ockham's program as they understood it. The clues appear especially in those texts that involve the understanding of relation and relative concepts. The problem is whether a relation is internal or external to the thing in relation. If internal, then the relation could be construed as an accident of the thing in relation. If external, then the relation could be construed as referring to the things in relation and to a description of the relation without positing some new entity in the things in relation.[63]

The Parisian natural philosophers were not in complete agreement among themselves. Here we focus on the views of Nicholas Oresme and John Buridan.[64] Of the Parisian natural philosophers, Oresme was the one who departed least from Ockham's view of connotation, yet he too found his theory, as he understood it, too lean. In his questions on the *Physics*, Oresme focused on the definition of motion as "*aliter se habere quam prius.*"[65] This was a definition that Ockham regarded as identical with his own.[66] Oresme, like Ockham, rejected the explanation of *motus* as requiring an inhering *res super-*

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[63] Cf. J. Brower, "Abelard's Theory of Relations: Reductionism and the Aristotelian Tradition," *The Review of Metaphysics*, 51 (1998), 605-631; and M. Henninger, *Relations: Medieval Theories 1250-1325* (Oxford, 1989).

[64] For the latest revisions on Oresme's academic career, see W. Courtenay, "The Early Career of Nicole Oresme," *Isis*, 91 (2000) 542-548. The contrary reactions to Ockham's views mentioned by Courtenay and documented by him and others may have motivated Oresme to adopt a reading of Ockham midway between critics and supporters. Albert of Saxony was also influenced by Ockham, but he seems closer to Buridan than to Oresme in his reaction to Ockham. On Albert's Ockhamism, see J. Sarnowsky, *Die aristotelisch-scholastische Theorie der Bewegung, Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters*, Neue Folge, 32 (Münster, 1989), 60-64, 91-99, 281-282, 316, and 416.

[65] See S. Caroti, "Oresme on Motion (*Questiones super physicam*, III, 2-7)," *Vivarium*, 31 (1993), 8-36, esp. 10, 18-21, and 27-33. Compare with Caroti, "La position de Nicole Oresme sur la nature du mouvement (*Questiones super physicam* III, 1-8): problèmes gnoséologiques, ontologiques, et sémantiques," *Archives d'histoire doctrinale et littéraire du moyen âge*, 61 (1994), 303-385.

[66] Guillelmus de Ockham, *Summula Philosophiae Naturalis*, ed. S. Brown, *OP*, VI (St. Bonaventure, 1984), 254[41-45]: ". . . Est autem ista descriptio eadem realiter cum illa qua dicitur quod 'mutari est aliter se habere nunc quam prius': non enim aliter aliquid se habet nunc quam prius nisi quia habet formam vel locum quem prius non habuit vel caret forma vel loco quem prius habuit." Also cited by Caroti, "Oresme on Motion," 10, n. 8.

addita or *distincta*.[67] Oresme, however, rejected Ockham's elimination of an internal reference mark in the *mobile*, and interpreted 'motion' as a *res successiva*.[68] At the same time, however, Oresme also denied the inherence of the *res successiva* in the mobile.[69] Oresme interpreted the *res successiva* as a "*modus seu conditio ipsius mobilis* "while denying of it that it is a *res superaddita*. Oresme's theory, in short, was clearly a compromise between the leaner reading of Ockham adopted by Ockham's critics and some more robust view that regarded the *res successiva* as an accident or accidental form, that is, an internal reference mark that is a disposition different from the *mobile* and inhering in it.[70] Oresme held the view that there is an internal reference mark but not an inherence in the thing. What is the *res successiva*? It is a mode or condition of the *mobile*.[71] From a logical point of view, Oresme adopted an interpretation of *motus* and *modus rei* that can be expressed only in a proposition (a *complexe significabile*), not by a term (an *incomplexe significabile*).[72] This was an interpretation that resembles Ockham's view that connotative terms and the propositions in which connotative terms appear require translation into complex propositions that describe or summarize two or more facts and conditions or some relationship that the terms signify. Oresme was clearly struggling, as Ockham had, with a way of affirming the reality of motion without positing an entity distinct from the *mobile*.[73]

The more inflationary view rejected by Oresme was Buridan's. Even on mathematics Buridan adopted a view different from Oresme's. It is true that Buridan regarded the terms referring to mathematical

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[67] Caroti, "Oresme on Motion," 10 and 32.

[68] Caroti, "Oresme on Motion," 28.

[69] Caroti, "Oresme on Motion," 28.

[70] Caroti, "Oresme on Motion," 28. Cf. Courtenay, "Early Career," 547, for some likely candidates. That their concerns, however, were not completely misdirected is suggested by Nicholas of Autrecourt's use of Ockham's principles. See B. Dutton, "Nicholas of Autrecourt and William of Ockham on Atomism, Nominalism, and the Ontology of Motion," *Medieval Philosophy and Theology*, 5 (1996), 63-85.

[71] Caroti, "Oresme on Motion," 29.

[72] To my knowledge, Stefano Caroti was the first to recognize the importance of the Parisian interpretation of a *modus rei* and *dispositio* as a *complexe significabile* specifically as a strategy to avoid a *res* -theory like Buridan's. See Caroti, "Oresme on Motion," 27-36. See also S. Caroti, "La perception du mouvement selon Nicole Oresme (*Questiones super physicam* III, 1)," in *Comprendre et maîtriser la nature au moyen âge, Mélanges d'histoire des sciences offerts à Guy Beaujouan* (Geneva / Paris, 1994), 83-99, esp. 90 and 98-99.

[73] The use of 'mode' suggests the view held by Henry of Ghent, but Oresme rejected the view that motion is a real accident. Cf. Henninger, *Relations*, 52-56 and 180-181.

entities as connotative; nevertheless, the conclusion that he drew eliminated mathematics from natural philosophy because the two sciences operate, as it were, on different levels of being. As a consequence, mathematics cannot help us to explain the reality of *res* in natural philosophy. Buridan may have believed that his interpretation was closer to Aristotle's intention, but it is hard to imagine a conclusion that demonstrates the extent of Buridan's divergence from both Ockham and Oresme more clearly than Buridan's restriction on the usefulness of mathematics in natural philosophy.[74]

Some Parisians were uncomfortable with Ockham's reductivism, so that as they followed him in his theory of connotation, they could not follow him in what they evidently took to be an eliminative reductionism. As a consequence, the Parisians conceived of motion, for example, as a mode (Oresme) or accident (Buridan) of being. In logic, however, Oresme adopted Ockham's characterization of terms like 'motion' as connotative, and like the Mertonians, he followed him in rejecting artificial constraints dictated by ontological considerations on the application of mathematics to problems of change and variation.

Before we elevate that observation to the level of a general conclusion, however, it behooves us to reflect more broadly on Oresme's project in natural philosophy. In all of the edited texts that deal with quantitative concepts and concepts of motion, change, place, and time, Oresme was abundantly clear in taking them as connotative.[75] In the cases where he diverged from Ockham, his solution was clearly a compromise between Ockham's view and a more realist interpretation. Even on *species*, perhaps the most notorious example, Oresme adopted a moderate view that displays some Ockhamist influence.[76]

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[74] See J. M. M. H. Thijssen, "Buridan on Mathematics," *Vivarium*, 23 (1985), 55-78, esp. 74-77. Aristotle used mathematical examples extensively throughout his works in natural philosophy, and in the middle sciences (optics, harmony, and astronomy) he even maintained that knowledge of the *cause* is sometimes the business of the *mathematician*. See *Analytica Posteriora*, I. 13. 78b32-79a13. Cf. T. Heath, *Mathematics in Aristotle* (Oxford, 1949), 59-61.

[75] See S. Kirschner, *Nicolaus Oresmes Kommentar zur Physik des Aristoteles, Sudhoffs Archiv*, 39 (Stuttgart, 1997), 197-198, 208, 213, 216, 218, 220, 227, 228-229, 296, 308, and 369-371. Compare with Nicole Oresme, *Quaestiones Super de Generatione et Corruptione*, ed. S Caroti (Munich 1996), 83*, 114*-115*, 4-6, 11, 53, 80, 96, 113-114, and 159. Finally, see B. Patar, *Nicolae Oresme Expositio et Quaestiones in Aristotelis de Anima, Philosophes médiévaux*, XXXII (Louvain-la-Neuve / Louvain / Paris, 1995), 387-388.

[76] Ockham, *Quaestiones in Librum Tertium Sententiarum (Reportatio)*, *OT*, VI, ed. F. Kelley and G. Etzkorn (St. Bonaventure, 1982), q. 2: "Utrum sensibile imprimat

As in his account of motion, Oresme denied not only the material or corporeal nature of *species*, but denied of them any reality altogether in the sensible world. Oresme consistently referred to *species sensibilium* never *species sensibiles*.[77] He denied the existence of *species in medio* while rehabilitating them as purely intelligible considered as perceived. The spiritual or immaterial interpretation could derive from Thomas Aquinas or the perspectivists who also denied the material nature of *species* while supposing them to be transmitted or carried through a medium.[78] In Oresme's theory there is no physical alteration of the medium, yet *species* do have a spatio-temporal reference.[79] This interpretation correlates Oresme's ideas on configurations of

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speciem suam in medio realiter distinctam ab eo." The full text takes up pp. 43-97. The following are exemplary: pp. 59-60: "Primo, quod ab obiecto visibili non causatur in medio aliquid alterius rationis ab ipso. . . Primum probatur, scilicet quod a visibili, puta colore non causatur aliquid alterius rationis sed eiusdem, puta color causatur in medio a colore in obiecto, licet imperfectior, quia pluralitas non est ponenda sine necessitate. Sed nulla apparet necessitas ponendi tales species productas in medio alterius rationis ab obiectis a quibus causantur, quia cum istae species non possint sentiri ab aliquo sensu, non debent poni nisi propter rationem deductam ex principiis per se notis vel experimento. . . Sed probatum est supra quod aliquid potest agere in extremum distans, nihil agendo in medio." The last reference is to p. 48: ". . . probo quod non semper movens immediatum est simul cum moto, sed quod potest distare. Primo, quia sol causat lumen hic inferius iuxta terram et non medium illuminatum quod est inter solem et lumen causatum hic inferius prope terram." And the conclusion of that argument is on p. 50: "Igitur istud lumen intensum non immediate causatur ab alio medio illuminato, igitur immediate causatur a sole, et per consequens potest sol immediate agere in distans." For analysis of Ockham's unique view of species, cf. A. Goddu, "William of Ockham's 'Empiricism' and Constructive Empiricism," in *Die Gegenwart Ockhams*, ed. W. Vossenkuhl and R. Schönberger (Weinheim, 1990), 216-223, and idem, "William of Ockham's Arguments for Action at a Distance," *Franciscan Studies*, 44 (1984), 227-244.

[77] See the analysis by C. Gagnon in Patar, *Nicolai Oresme Expositio*, 139*-161*, esp. 154*-158*.

[78] On Thomas Aquinas, see *Summa Theologiae*, 1a, 85, 2 and 2, ad 1; *Quaestiones Quodlibetales*, VIII, 4. On the perspectivist tradition, see D. Lindberg, *Theories of Vision from Al-Kindi to Kepler* (Chicago, 1976).

[79] Cf. the study by Patar in Patar, *Nicolai Oresme Expositio*, 161*-170*. See *Nicole Oresme and the Medieval Geometry of Qualities and Motions, Tractatus de Configurationibus Qualitatum et Motuum*, ed. and tr. M. Clagett (Madison, Wisconsin, 1968), 166-168: "Ideoque intensiones equales per equales lineas designantur et dupla intensio per duplam lineam et sic semper proportionaliter procedendo. Et istud est universaliter intelligendum de omni intensione ad ymaginationem divisibili, sive sit intensio qualitatis active sive non active, sensibilis sive insensibilis subiecti aut obiecti aut medii, ut de luce corporis solis et de lumine medii, vel de specie in medio, vel influentia aut virtute diffusa, et sic de aliis, excepta forsitan intensione curvitatis, de qua dicetur ad partem in capitulis 20^0 et 21^0 huius partis. Huiusmodi vero linea intensionis de qua nunc dictum est non extenditur extra punctum vel extra subiectum secundum rem sed solum secundum ymaginationem, et ad quamvis partem nisi quod convenientius ymaginatur in sursum perpendiculariter stare super subiectum qualitate informatum."

qualities with his view of *species* by rendering *species* as intelligible in purely mathematical terms.[80]

Oresme's theory is clearly not as reductive as Ockham's yet it shows sensitivity to Ockham's critique of inflationary accounts. Oresme treated *species* in a way analogous to his treatment of 'motion' and *res successiva*. Finally, his use of mathematics was consistent with the interpretation of *species* as spiritual and dimensional configurations. In his interpretation of connotation and its application to *species* and mathematics, Oresme among Parisian natural philosophers came closest to following Ockham.

General Conclusions

This is far from saying that either Ockham or Oresme adopted mathematical analysis in the modern sense. The point of this interpreta-

[80] *Nicolae Oresme Expositio*, III, q. 10, 388: "Sed aliquando hoc nomen *motus* per aequivocationem accipitur pro re acquisita, connotando quod ipsa de facto acquiritur. Et sic dicit ARISTOTELES quod est de genere termini ad quem; et isto modo actus intellectus est idem quod illa qualitas vel similitudo et species, sicut dicimus quod alteratio est qualitas acquisita." Ibid., 389-390: "Quarta conclusio est quod habitus est illa eadem similitudo quae vocatur 'species', connotando tamen maiorem firmitatem et etiam quod inclinat intellectum, quia non quantum libet intensa potest intellectum inclinare. Patet ergo recapitulando quod eadem est qualitas quae, dum acquiritur, vocatur 'actus', et dum intellectus cognoscit et quando non, tunc solum, vocatur 'species', et quando est bene firmata et inclinans, tunc vocatur 'habitus'. Et cum hoc etiam actus bene potest accipi pro fluxu successivo, ut dictum est. Sciendum etiam quod, si ponerentur qualitates distinctae, per alium modum non posset demonstrative improbari; tamen non videtur rationabile quod ponatur talis multitudo sine ratione coagente, et in casu ubi sine, hoc est opinio magis probabilis et magis verisimilis. Et inveniuntur exempla in aliis propositis magis notis, sicut videmus quod eadem est gravitas quae, dum acquiritur, vocatur 'alteratio' et 'motus', et quando est parva, nundum potest inclinare perfecte ad motum deorsum; et quando est bene intensa, tunc potest. Et conformiter in proposito." To the fourth objection (384): "Quia caliditas non est res distincta a calefactione; immo motus est de genere termini ad quem, ut habetur V^{0} *Physicorum*; ergo similiter motus intellectus qui est actus eius non distinguitur ab illa qualitate quae dicitur species vel habitus", he responds (390): "Ad quartam concedo quod, sicut caliditas est calefactio, dum acquiritur, ita etiam species est actus, nisi actus accipiatur pro fluxu." To the sixth and seventh objections (385): "Quia intellectus agens est potens omnia facere et intellectus possibilis omnia fieri supponendo per cognitionem; ergo praeter ista nihil aliud requiritur, neque tamquam agens neque sicut patiens. Et sic talis species frustra poneretur et nihil iuvaret. . . Sequeretur quod talis species prius intelligeretur quam aliquid aliud, cuius oppositum experimur. Consequentia patet, quia primo obiiceretur et praesentaretur ipsi intellectui", he responds (390): "Ad sextam concedo quod intellectus agens est potens omnia, etc., non tamen per se solitarie; et etiam species non ponitur sicut agens aut passum, sed sicut effectus. Ad septimam dico quod species non intelligitur nisi reflexe; sed est illud quo aliud intelligitur, scilicet obiectum."

tion has been to show that Ockham's theory of connotation and his conception of mathematics influenced some Mertonians and Nicholas Oresme. Under Ockham's influence they adopted views that modified Aristotelian theory in ways that provided logical support for a different understanding of the role of mathematics in natural philosophy. The re-interpretation of disciplinary relations and disciplinary boundaries constituted a condition for the serious consideration of alternatives to the Aristotelian picture of the cosmos. That result was Ockham's most important contribution to late medieval philosophy of nature. The problem-oriented approach and questions helped to preserve the core of Aristotelian philosophy of nature while at the same time exposing individual problems to closer and more precise scrutiny.[81]

The problems confronted by medieval thinkers in astronomy, theories of light and vision, music, compound medicines, mixtures, and many other disciplines demanded more exact treatment. Despite their efforts, however, the Mertonians and Oresme corrected the problems of motion in ways that left most of the basic principles of Aristotelianism intact. The Copernican theory revised the relationship between mathematics and natural philosophy as providing the warrant for conclusions about the structure of the universe.[82] No urgency existed for the resolution of the problems of falling bodies, for instance, until Copernicus's revision achieved plausibility several decades after its publication in 1543. Furthermore, even when the geometrical analysis was applied to bodies falling freely from rest, producing experiments and using the results to confirm the mathematical analysis required extraordinary virtuosity in addition to mathematical ability. Measurable results could be obtained only by retarding the speed of a falling body and then extrapolated to the motion of bodies falling freely from rest. The problems of interest to the Mertonians and to Oresme remained broadly consistent with Aristotelian natural philosophy and cosmology. That is, the problems were formulated in terms of characteristics of nature apprehended directly by the senses, of quantitative analysis subordinated to a fundamentally qualitative approach to nature, of causal powers

[81] See J. M. M. H. Thijssen, "Some Reflections on Continuity and Transformation of Aristotelianism in Medieval (and Renaissance) Natural Philosophy," in *Documenti e studi sulla tradizione filosofica medievale, Rivista della società internazionale per lo studio del medioevo latino*, II, 2 (Spoleto, 1991), 503-528, esp. 518-519.

[82] See R. S. Westman, "The Astronomer's Role in the Sixteenth Century: A Preliminary Study," *History of Science*, 18 (1980), 105-147, esp. 107-116.

and resisting media, and of local motion in an earth-centered cosmos.[83]

Ockham's critique of the realists was effective, but it failed to be completely convincing. Accordingly, his critique led to compromise views that struggled with questions about the reality of categories derivative to substances and qualities. Some of the struggles had to do with the representation of change and variation. Freed by Ockham from artificial metaphysical constraints and rigid disciplinary boundaries, the Mertonians and Oresme employed mathematical methods and figures to represent change and variation. Seen from this perspective, the developments in natural philosophy from the late Middle Ages to the early modern period were less new and less surprising than seventeenth-century revolutionaries would later proclaim. The increasing application of mathematics to problems in natural philosophy, the implicit (sometimes explicit) proposals to make mathematics and mathematical principles superior to the principles of natural philosophy, and the willingness to represent mathematical explanations as causally explanatory—all of these developments were not especially shocking to those who had followed a more Ockhamist reading of Aristotelian natural philosophy.[84] The Cartesian decision to employ magnitude and figure in the representation of physics was also not surprising. On the other hand, what does remain surprising was Descartes' insistence that physical things could be represented by nothing other than magnitudes and figures.[85] Be that as it may, Ockham's influence on modern philosophy, mediated by the Mertonians, Oresme, and Francisco Suarez, can be seen

[83] Indeed, it is likely that Copernicus himself intended his theory—insofar as it touches on natural philosophy—as a challenge to Aristotelians to adapt Aristotelian principles to a heliocentric system. I am preparing a comprehensive study of Copernicus's relation to the Aristotelian tradition. See N. Swerdlow, "Copernicus, Nicolaus (1473-1543)," in *The Encyclopedia of the Scientific Revolution*, ed. W. Applebaum (New York / London, 2000), 162-168, esp.164-165.

[84] I have in mind the startling comments by Galileo in the *Discorsi*, in *Le opere di Galileo Galilei*, ed. A. Favaro (Florence, 1898), VIII, 296, about a mathematical demonstration providing a causal explanation. Cf. the remarks by Newton in the "Scholium Generale" of Book III, *Philosophiae Naturalis Principia Mathematica*, ed. A. Koyré and I. B. Cohen (Cambridge, Massachusetts, 1972), II, 759-765, where he speaks of the mathematical principles as if they were physical causes.

[85] Compare the following accounts: D. des Chene, *Physiologia: Natural Philosophy in Late Aristotelian and Cartesian Thought* (Ithaca / London, 1996), 119-121; A. Robinet, "Dialectiques et regulae: lieux et concepts," in *Descartes et le moyen âge, Études de philosophie médiévale*, 75, ed. J. Biard and R. Rashed (Paris, 1997), 231-240, esp. 237-238; and C. Sasaki, "Descartes as a Reformer of the Mathematical Disciplines," ibid., 37-45.

most clearly in the ideas of Leibniz and Kant on the transcendental character of quantity and relation.[86]

This essay concludes with emphasis on the ironic consequence of Ockham's critique of inflationary accounts. As he liberated mathematics from ontological constraints, he freed the imagination to treat mathematical entities as if they did exist.[87] The logical purists could always reject ontological commitments as confusions, while the less logically cautious could easily fall into the very confusions that he had tried to dispel. As Curtis Wilson, making a different point, so insightfully concluded:[88] "Mathematical analysis, in the seventeenth and eighteenth centuries, seems to have been too busy with its astonishing empirical triumphs to be concerned with logical rigor." If we regard these developments in mathematical analysis from the practical perspective of natural science, what scientists and mathematicians did with the tools developed is more important than what they believed about them.

Appendix

Here are examples where the modern edition of Ockham's *Expositio* inserts a word or phrase that corresponds with the translation in Moerbeke's final version (indicated by 'G') for a translation that corresponds with that of James of Venice (indicated by 'V'), and examples where the version in V is used instead of the version in G. For the examples, I relied on Brams and Vuillemin-Diem, pp. 245-251, using their Bekker numbers for reference.[89] All of the texts of the *Expositio* are from *Opera Philosophica*, V.

1. (229b14-22): the use of *extremum* (G) for *ultimum* (V) at *Exp.*, V, 8, 4^{3-9}.
2. (225b12): *verificari* (G) for *verum esse* (V) at *Exp.*, V, 3, $2^{171-172}$.
3. (229a24): *mutatio* (V) instead of *semotio* (G) at *Exp.*, V, 8, 2^{49-76}.
4. (226a31): *augmentatio* (G) for *augmentum* (V) at *Exp.*, V, 4, 2^{9}.

[86] See Beckmann, "Ockham," 87-89. For confirmation of the influence of Ockham's theory of connotation on Suarez, see S. Menn, "Suárez, Nominalism, and Modes," in *Hispanic Philosophy in the Age of Discovery, Studies in Philosophy and the History of Philosophy*, 29, ed. K. White (Washington, D.C., 1997), 226-256, esp. 232-242.

[87] "Freeing the imagination" attributes to Ockham a historical role that fits into a recent account of the rise of modern science. See A. Crosby, *The Measure of Reality, Quantification and Western Society, 1250-1600* (Cambridge, 1997).

[88] See Wilson, *William Heytesbury*, 114, and his conclusions, 148-150.

[89] Cf. Brams and Vuillemin-Diem, *Physica Nova*.

But note the variant *augmentum* in 2 MSS.

5. (225a16-17): *fieri* (V) instead of *generari* (G) at *Exp.*, V, 2, 1^{28}.
6. (226b28-29): *deficio* (V) instead of *interrumpor* (G) at *Exp.*, V, 5, 4^{9-16}.
7. (228b6): *intercipitur* (G's intermediate version) for *recipitur* (V) at *Exp.*, V, 2^{15}.
8. (230a32): *fatate* (G) for *moderate* (V) at *Exp.*, V, 10, 2^{16}.
9. (228b21): *recta* (G) for *rectitudo* (V) at *Exp.*, V. 7, 4^{16}.
10. (230b26): *stare* (V) instead of *statum ire* (G) at *Exp.*, V, 10, 5^{37}. But note the variant *statum ire* in one MS.
11. (226a15): *doctrina* (V) instead of *disciplinatio* (G) at *Exp.*, V, 3, 6^{40}. Note that at *Exp.*, V, 6, 1^{43-49}, the editors have *doctrinatio*, which Moerbeke used at *Metaphysica* 1069b15.
12. (225a13, 225b12): *mutor* (G) for *muto* (V) at *Exp.*, V, 2, $1^{22-24, 55-57, 61-62, 70, 77-93}$; *Exp.*, V, 3, $2^{6 \text{ and } 41-50}$.
13. (225b28): *egrotaverit* (G^2) for *infirmetur* (V, G^1) at *Exp.*, V, 3, $4^{61 \text{ and } 71-74}$.
14. (224b8 and 230a11-17): *esse* (V) instead of *ens* (G) at *Exp.*, V, 1, 4^{6}; and *Exp.*, V, 9, 3^{26-40}.
15. Yet (225a15): *ens* (G) for *esse* (V) at *Exp.*, V, 2, 1^{44}. But one MS. has *esse*.
16. (227a17): *insertus* (V) instead of *adnascentia* (G);
    (227a23): *insertus* (V) instead of *symfusis* (G);
    (227a27): *consertus* (V) instead of *symfusis* (G); all at *Exp.*, V, 5, 9^{33-38}.

Yet,

17. (227a25): instead of *apta nata sunt* (V) and *adnata sunt* (G), *Exp.*, V, 5, 9^{42-63}, uses a form of *adunatio*, which seems to come from Moerbeke's first translation of *Generatio animalium*, 773a4.
18. (227b14): *opinio* (V) instead of *existimatio* (G) at *Exp.*, V, 6, 1^{48}.
19. (226a33, 227b5, 228a29, and 230b13): *loci mutatio* (V) instead of *latio* (G) at *Exp.*, V, 4, 2^{37}; *Exp.*, V, 6, 1^{12}; *Exp.*, V, 7, 1^{69}; and *Exp.*, V, 10, 3^{23}.
20. (229a2 and 229b7): *motus* (V) instead of *latio* (G) at *Exp.*, V, 7, 5^{2-8}; and *Exp.*, V, 8, 3^{65-92}.

ABSTRACT

This article summarizes Ockham's interpretation of Aristotle's categories, showing how his account of connotative concepts introduced a revision in the Aristotelian doctrine about the relation between mathematics and physics. The article